# Developing firm mathematical foundations for all attainers

**Carol Handyside** uses the 5 dice pattern to support subitising and visualising

You may know the story of the young seven-year old who, when asked to add all the integers from 1–100, instantly solved the problem. Those who have heard of Carl Friedrich Gauss know that he went on to be one of history's most influential mathematicians. His method was based on an acute awareness of the patterns of numbers within 100. He used reasoning to deduce that if he added 1 and 100, 2 and 99, 3 and 98 etc, it would result in 50 pairs of numbers, each summing to 101, so the total of these pairs would be 5050.

The significance of Gauss's achievement was not in his ability to perform a given method, but that he discovered it for himself. It also illustrates the power of patterns, flexible thinking and reasoning in the field of mathematics. It shows that when a child knows all their numbers from 1 to 100, it does not mean that they need to be rushed onto learning higher numbers. There are a multitude of patterns to explore before proceeding to higher numbers, firstly within 10 and only then generalising to 20, 50 and then 100 etc.

Furthermore, there are many patterns to explore within single numbers. By drawing out a deep level of awareness of each number, children may be able to see links and make connections between numbers.

## Background

My teaching practice has shifted over the years and the more I work at planning for greater depth, the more I realise that this is the way that I love to teach. I have always felt passionate about teaching for understanding. I like finding ways for children to reason and make mathematical discoveries for themselves and I have always had an appreciation for the creative element of mathematics. It is very exciting to be teaching at a time when there is a spotlight on creativity and depth of understanding.

Maths is about logic and reasoning, patterns, seeing connections and problem solving. It is crucial that a child's first experience of maths should nurture these skills rather than focusing on memorisation of number facts. Rote learning has the potential to crush that desire to make logical sense of numbers and dampen problem solving and reasoning skills. Research has also shown that once students have memorised procedures without first understanding, it is difficult to get them to make sense of them later (McClure, 2014).

For the past ten years, much of my work has focused on teaching maths to lower attaining children of various ages, from lower primary to GCSE level maths. I have noticed a strong lack of number sense and understanding in lower attainers, even though most of the children I have worked with have a good aptitude for reasoning.

I saw the need for something that would help children make sense of numbers and give them a strong visual representation of numbers that linked with their fingers. I also noticed that children who lacked number sense relied heavily on counting; did not easily make connections in their learning and had an overreliance on formal written methods. This, together with issues around teaching formal methods too early and the problems that arise when children do not use known number facts flexibly has been researched extensively (Gray and Tall, 1991), (Boaler, 2008).

I have seen far too many intelligent children not reaching the GCSE maths exam level of 4 grade or above as a result of not acquiring basic number skills – more than a quarter of 16-year olds. Without basic number skills early in their schooling, they become left behind year after year and the gap becomes wider. Students who do not achieve a 4 grade or above in maths may find many college courses and certain careers off limits. 'Numeracy problems have a stronger negative impact on job prospects than literacy problems, throughout an adult's life' (Parsons and Bynner, 2005).

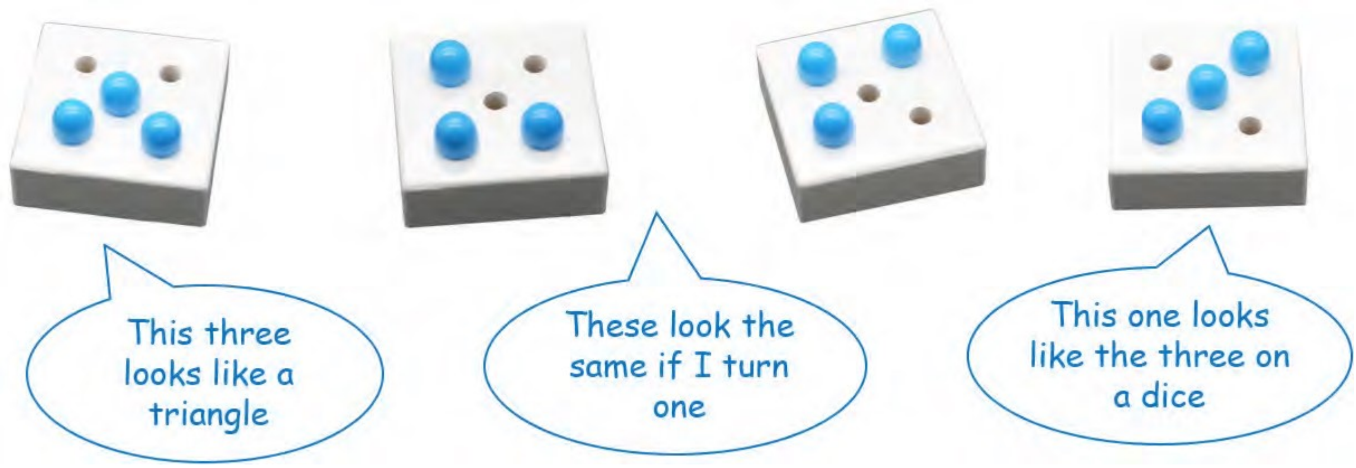
I felt so compelled to help teachers create a classroom where every child excels at maths,

that I have spent every moment of my spare time completely devoted to developing Spot On With Numbers. It is a tool for creative maths teaching, where children can make and make sense of numbers, discover patterns, reason, work flexibly with numbers and remove the over reliance on counting. If children are given the time to explore numbers with even greater depth, so that they can make discoveries around patterns and efficiencies, they may go on to be the next most influential mathematicians in history. At the same time, children who do not see the links as quickly have time to make sense of numbers and learn to work flexibly with numbers. It may be that some children need to spend a little longer making and understanding numbers so that they do not get left behind.

## Many ways to make and explore numbers

Spot On With Numbers is based on the five dice pattern as it links to the fingers. Fingers are the first place wherechildren make sense of numbers. In an effort to reduce the over reliance on counting, the five dice pattern can be used to subitise and visualise.

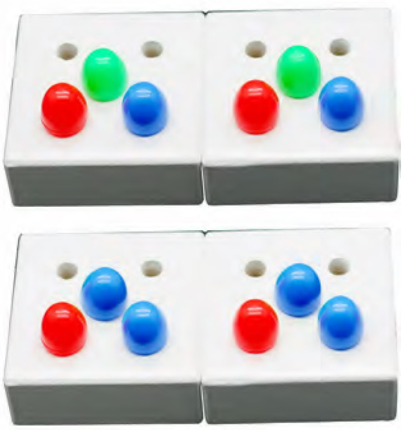
I wanted to give teachers, LSAs and parents an alternative resource for when children are not understanding maths in the way it is being presented. I appreciate that the ten frame has been very successful in reducing the over reliance on counting and providing a concrete and pictorial representation for numbers. I also fully appreciate the concern over confusing children with too many manipulatives. However, in working with children, I have seen the light bulb moments when children make connections between multiple representations using different concrete resources. Once children have creatively explored many representations of a number and are encouraged to discuss the similarities and differences, they acquire an even deeper understanding of the number.



The five dice pattern gives children another way of seeing numbers. It allows for multiple representations and links with familiar patterns. The ability to move a peg and see that the number stays the same even though the configuration has changed, gives children an understanding of conservation. The diagonal encourages us to see a set of pegs in more arrangements, similar to those which could occur in a random collection. It can therefore be used as a scaffold for subitising. If children explore all the representations of, for example three, and are able to explain what they notice and see the links to familiar patterns, they will be more confident at subitising three objects in a random arrangement.

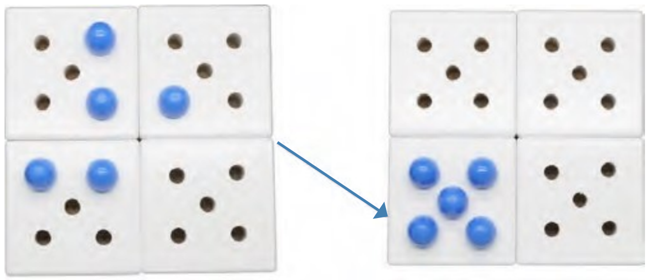
## Patterns

If three is made with different colours, there is the opportunity for exploring, for example ABC and ABB patterns as well as noticing that three can be made up of 2 and 1.



The composition of all numbers (for example five in the picture below) can be explored as a pattern.

The ability to move the pegs into a familiar pattern also strengthens the link between the random arrangement and the familiar arrangement of the number five.

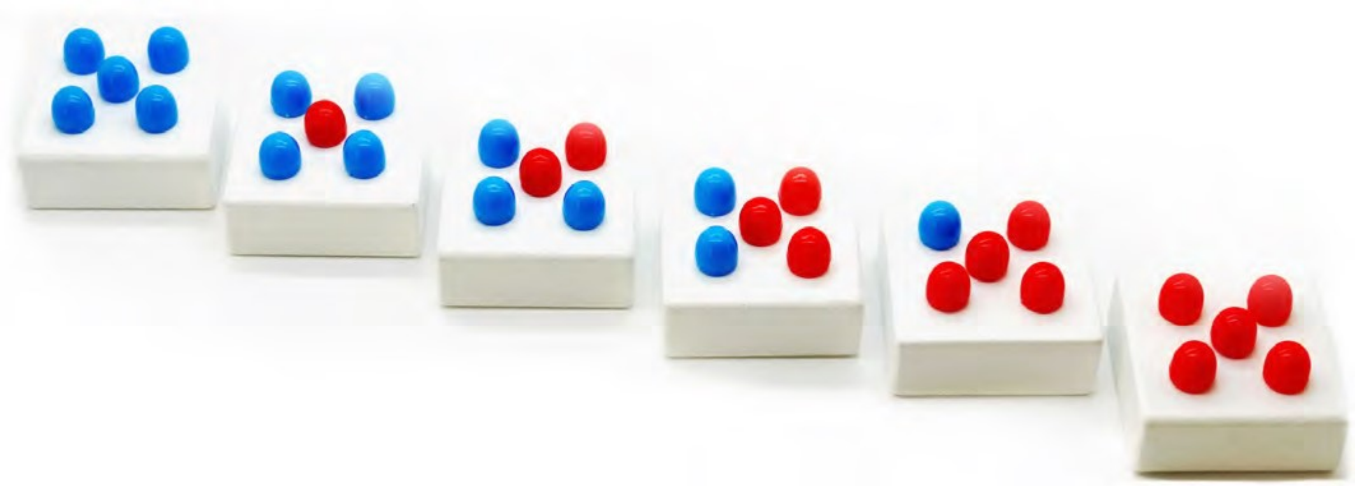


The numbers that make up five, for example, can also be explored by using different colours (e.g. two green, two blue and one orange to show how five is made up).

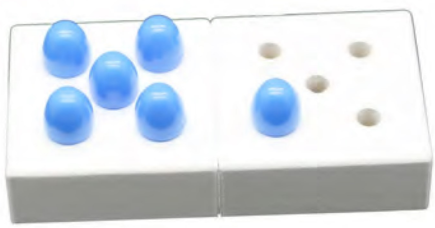
## Subitising and number sense

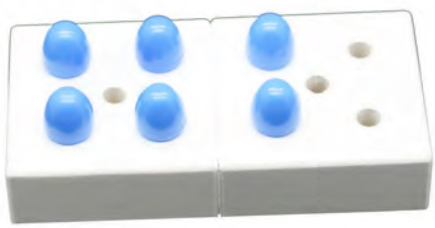
By the age of five, children may be ready to subitise up to five objects (NCETM, 2018).

It is commonly known that children who do not acquire the ability to subitise also have difficulties in maths. The capacity to subitise is apparent in chimpanzees (Murofushi, 1997) and probably came before counting in the evolution of man. It is most interesting that some children do not naturally develop a skill that is most likely innate. Work by David Mills has shown that 'subitizing ability in dyscalculic children could be improved into the normal range' and 'their math skills… had also significantly improved' with their ability to subitise (Mills, 2015). Mills' work shows dyscalculic children can be taught to subitise. It highlights the importance of teaching subitising alongside counting.



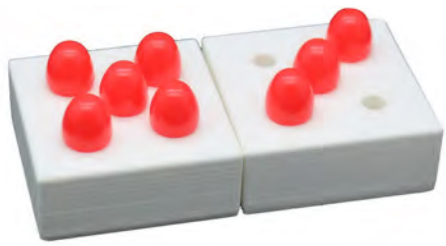
If children can develop an ability to subitise five objects, a greater number of objects can be subitised by using known number facts to instantly recognise the total number of objects. The five dice formation allows children to constantly reinforce these number facts. For instance, 5 + 1 = 6 and 4 + 2 = 6. By moving the pegs, children see the links between these number facts and if they can communicate what they have done, it strengthens their reasoning skills.





Subitising plays an important role in developing number sense as it encourages children to see how numbers are composed and partitioned and the child develops a sense of the number. To conceptually subitise 8 in the configuration above right requires knowing that 8 is composed of 5 and 3, but it is also 4 and 4 (or double 4).

In both examples, it is clear that 8 is 2 less than 10 as there are 2 empty holes. By playing with colours and discovering that 8 is also composed of 6 and 2; knowing that 8 is one more than 7 and one less than 9 by physically adding and removing pegs; all combines to give an 8-ness of eight, thereby developing number sense. If children can see the numbers 7 and 8 without having to count each set

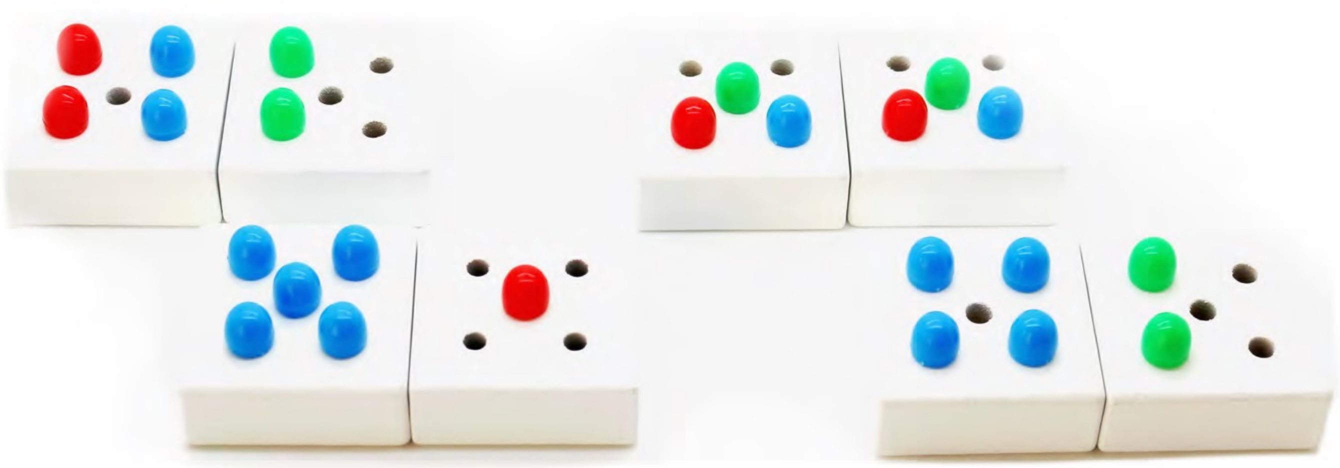


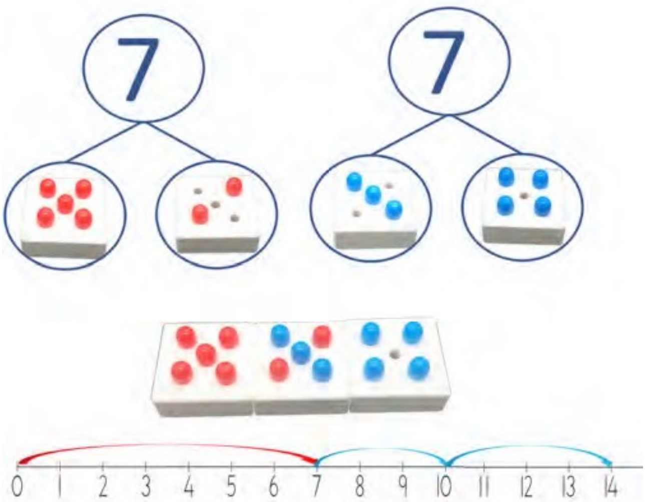
of the components in turn, the links become much clearer. When children can see the links, for example, in this case that double 4 is 8, they develop fluency through visualising, seeing links or drawing on previous learning.

## Flexible partitioning and thinking

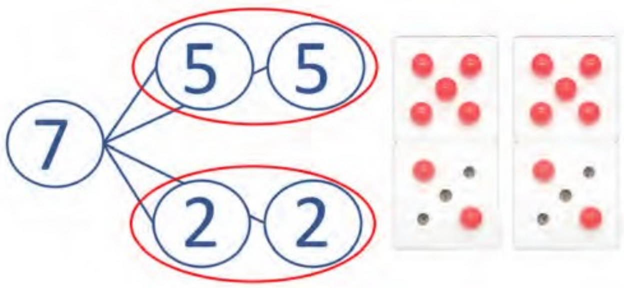
Fluency in mathematics is not just about the ability to perform a calculation accurately and efficiently. It is also important that it is performed flexibly. Early flexibility comes from being able to partition numbers into their component parts. Making any number greater than five with the pegs and boards has to be made with at least two parts, which encourages children to compose and decompose numbers. If children are making numbers creatively with any number of boards and different colours to represent component parts, this further develops this critical stage of development.

Knowing that, for instance seven can be composed in different ways, is important when bridging the tens. For example, when adding 7 and 7 together (see top of page 7):



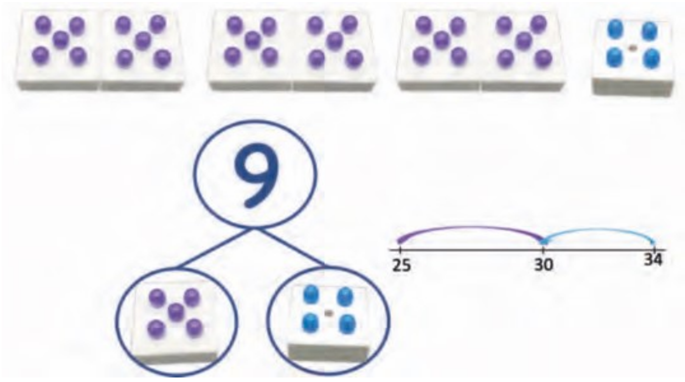



However, seeing 7 just as 5 + 2 provides a different way to efficiently double the number. Using the pegs and boards to make 7 creates an obvious partition of 7 into 5 + 2. If we regroup the numbers to make 10 + 4, we can see that the total is 14. This formation also provides a strong visual image for this number fact which could be used to aid memorisation.



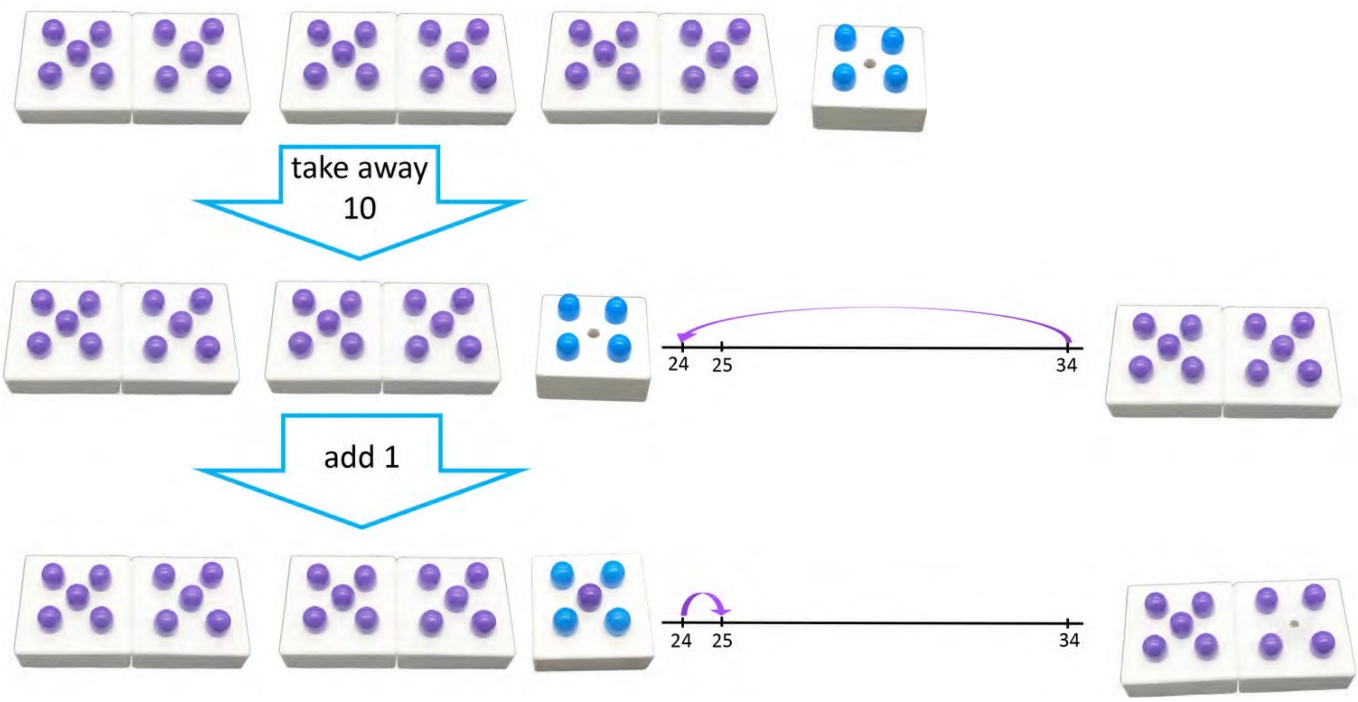

If children can appreciate and explain both methods of partitioning and doubling 7, they are laying the foundations to master the art of flexible partitioning.

An efficient mental strategy to subtract 9 from 34, may be to partition the number which is being taken away, in order to cross the tens boundary. Here, 34 takeaway 4 is 30 and then we need to takeaway 5 to reach 25.




This is an efficient strategy, but it is equally as efficient to first take away ten and then add one.

Relying on only one of the above strategies means that later, when children come to do more complicated calculations such as 347 – 99, they are more likely to rely on the formal algorithm they have been taught for subtraction. The pegs and boards offer a chance to explore multiple strategies and develop flexibility with more than one efficient method.



## The future

Our greatest mathematicians of the past showed flexibility, creativity, problem solving skills and the ability to consider multiple solutions to find efficiencies. The future requires even more competent mathematicians as the jobs of the future will require even more children to be confident with STEM subjects.

According to the Office for National Statistics (ONS), 7.4% of current jobs are at risk of being replaced by automation (BBC News, 2019). Technology is progressing at an exponential rate and maths teaching needs to reflect the skills required for future jobs. Computers are able to perform calculations, which means children do not need to learn to perform procedures or follow methods. We, therefore, need to educate children to be able to compete in a competitive global job market. They will require problem solving skills, creativity and critical thinking. This means thinking outside the box, considering multiple solutions and the ability to think flexibly (Crimson Education 2017). It is difficult to think about five- year olds in terms of their future careers, but it is here that we lay the foundations of mathematical thinking that they take with them into life.

## Conclusion

Spot On With Numbers provides children with a resource to see numbers slightly differently. It links to the fingers and encourages children to subitise. As a concrete resource that can be physically manipulated, there is not a reliance on counting. It offers an alternative way to explore patterns and partition numbers thereby allowing children to explore concepts in greater depth. As children are actively involved in their learning, they can use it to truly understand, reason, think flexibly and find multiple solutions. Used effectively, it can be successful in laying solid foundations for mathematical principles and thinking.

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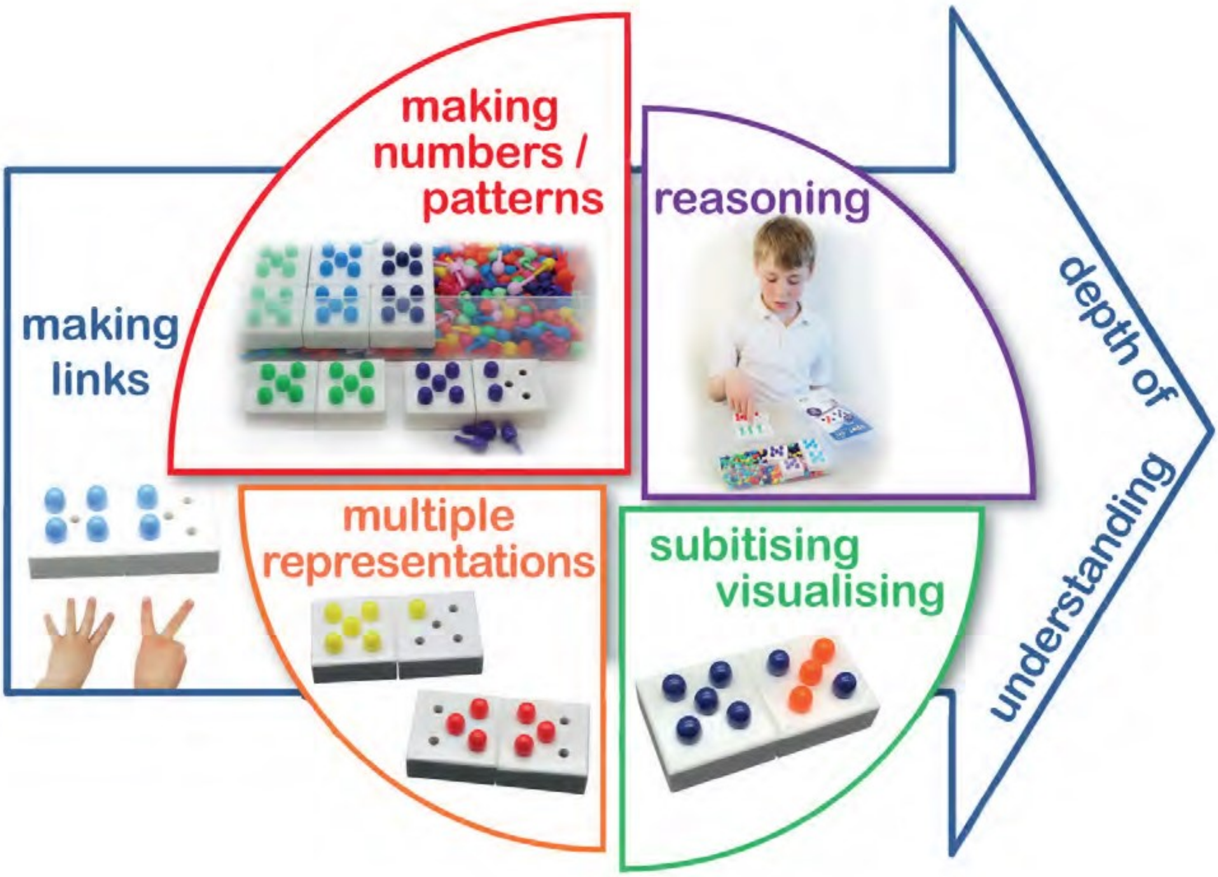

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**Carol Handyside** is a maths teacher and the developer of Spot On With Numbers. As a new product, there will be lots more to share, so please follow us on twitter (**@spotonnumbers**) or Facebook (**@spotonwithnumbers**) and browse our website, which does have a free resource, to enable you to try the five dice formation: **www.spotonwithnumbers.co.uk/free-resources**