Developing a deep sense of number for all children

Carol Handyside and **Louise Langford** share their use of dot patterns

Carol and Louise have both spent many years training and working as maths specialists. Despite taking different routes in mathematics education, they have recently collaborated due to their shared interest in developing the use of structured dot patterns as a tool to develop a deep sense of number for all children.

This is their story...

Louise

My fascination with 'dots' began several years ago when I had the opportunity to train as a Dyscalculia Specialist and then went on to complete a Masters, researching Early Mathematics Intervention. At the time, I was increasingly aware of the growing number of pupils with Mathematical Learning Difficulties (MLD), currently thought to impact 25% of pupils with 5% potentially dyscalculic (BDA, 2020), as well as the need to close the attainment gap for disadvantaged pupils. As my role then and now includes improving mathematics teaching, this fuelled my interest in researching strategies that could be employed not only in intervention but also in whole class teaching. I therefore went on to consider mathematics teaching and learning, from both a preventative and intervention perspective.

Those with a MLD or dyscalculia have often not mastered the concept of a number or number sense. They may need to actually count the dots on a domino, struggle to connect a numeral with an amount and have problems deriving and recalling number facts. These difficulties can all lead to computational weakness. Dyscalculia is a sub-type of MLD (SASC, 2019) and those with dyscalculia have very domain specific difficulties with magnitude comparison, ordering and subitising, that is, the ability to appear to suddenly 'count' or determine the number of objects in a small collection (Clements, 1999).

Dyscalculics really struggle to subitise more than 2 or 3 items, often having problems manipulating numbers visually in their heads to complete tasks such as sequencing numbers or applying patterns to reason mathematically. This can be a real difficulty as understanding and using patterns is central to

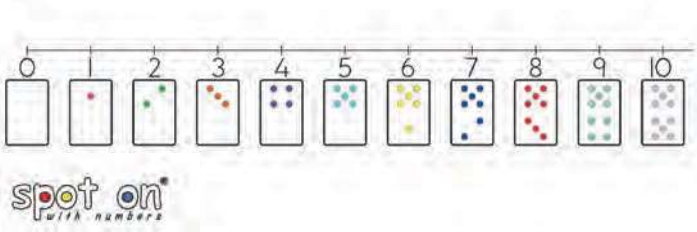
Carol

My interest with the structure and representation of number began in the year I trained as a maths specialist. When I think about how I can translate my understanding of maths, it naturally emerges as a picture or physical resource. So, when I first encountered students who I saw lacked number sense, dice patterns and fingers felt like the best starting point to secure an understanding of number.

We know that 'number sense is a foundation for higher-level mathematics and correlates with later achievement in maths' (Cambridge Mathematics, 2017). Methods of teaching to develop number sense, that have been proven to help children who have demonstrated poor number sense, will therefore be beneficial to all children. Furthermore, the structural variation used in these methods provides all children the opportunity to explore number at greater depth.

I designed *Spot On With Numbers* around the key elements of developing number sense. It is recognised that number sense is not something that can be explicitly taught, but is developed through exploring numbers, moving between representations, encouraging flexibility, creativity, visualisation and making connections. Key to visualising and perceiving numbers is the ability to subitise and an appreciation that numbers can be composed and decomposed into parts is crucial to working flexibly with numbers.

the number system and mathematical reasoning. Furthermore, the emergence of counting skills is significant in children's conceptual development as this supports number sense and lays a strong foundation for future learning.



Subitising

Louise

Most people can instantly recognise four items (Butterworth, 2019). This is known as perceptual subitising, where a small amount can be observed and remembered by linking the amount to its number set. Using grouping, larger amounts can then be subitised such as 4 and 3 make 7. This is known as conceptual subitising, where amounts are seen as composites of parts and as a whole (Clements, 1999). Understanding number as a set supports the key skill of estimation, developing mathematical reasoning through counting, comparing and grouping.

Interestingly, research suggests those with dyscalculia can learn to subitise and develop understanding of number sets, moreover, we know that infants can process number so there is a foundation on which to build (Reid, 2016). This led me to consider, how can we teach students to subitise, to see an amount in their head and connect to the numeral?

As part of a dyscalculia case study, I utilised multi-sensory dice dot cards to encourage subitising. Making the students own 'feely cards' with structured patterns and corresponding numerals, supported them in knowing the cardinality of a number whilst allowing them to explore number facts. We also developed finger patterns plus 'throwing numbers' to support conceptual subitising. Indeed, many experts in the field of dyscalculia such as Butterworth, Chinn and Sharma recommend using 'dice patterns' as a method of teaching as this enables the visualisation of a number that can be organised to facilitate retrieval and flexible application whilst reducing memory load.

This approach proved effective in supporting my students to visualise and internalise dice dot patterns, developing automaticity of key facts, improving estimation and understanding of the number system. This led to increased confidence and computational accuracy.

Carol

As a part of Louise's work as a PD Lead, I was lucky enough to connect with her to share both professional discussion and resources.

Similar to Louise, I have found finger patterns and helping these learners visualise, subitise and internalise an appreciation of numerosities through dice dot patterns very effective to develop a sense of number.

Although Spot On With Numbers is based on dice patterns, the pegs and boards are flexible and numbers can be represented in many different ways, so the creation of numbers is child led. The use of the iconic five pattern and the fact that any number lower than five can be subitised encourages and develops subitising skills. The use of pegs secured in the boards provides a resource where a created number can be moved and manipulated physically before moving on to the abstract way mathematicians mentally manipulate numbers.

Each representation of a number links to the fingers and can be manipulated to show the different variations, in the same way our fingers can compose numbers, as in the examples of 7.

This is one activity that can be used to develop conceptual subitising skills, alongside developing number sense as well as number bonds.

We know that the dice patterns are useful for learners with MLD. The Spot On With Numbers Pegs and Boards are BDA Assured as a tool for supporting learners with dyscalculia.

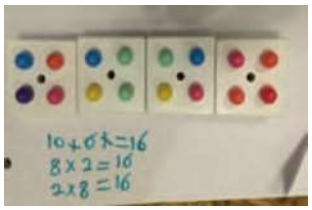
Our question is why these dot patterns are confined to specialist education, when we know all children can benefit from developing a deep sense of number. It is also surely better to teach to prevent MLD, using methods known to support these difficulties. To illustrate this, we aim to share our experience of teaching multiplication through effectively utilising structured dot patterns.

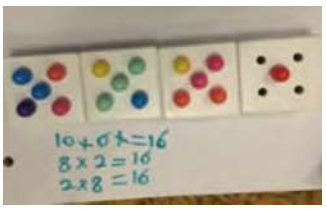
Multiplication

Louise

When teaching children to move from additive (repeated addition) to multiplicative reasoning, I have found the multiplication mat particularly useful (Emerson and Babtie, 2014). Using counters, potato printing or number frames to create groups supports children to step count initially. This gives an ideal opportunity to introduce the word 'multiples', spot patterns in multiples and link to other number sequences. Connections can be made between multiplication and division facts: if we know 2 × 10 = 20, what does this image show 20 ÷ 10 is? How many groups of 2 if we have 20 counters? The mats can be used to derive times table facts and encourage strategy, for example, if 2 × 10 = 20 then 2 × 5 = 10, as half of 10 is 5 and half of 20 is 10. Consider how this structure supports students thinking.

I used the 'Spot On' resources with the multiplication mat when working with a Year 2 student, initially looking at the five times table and moving from additive to multiplicative reasoning. We then explored strategies to derive and recall facts, for example, 6 lots is one more than 5 lots, 9 lots is one less than 10 lots. This worked really well in enabling them to reason, make connections and gain confidence.

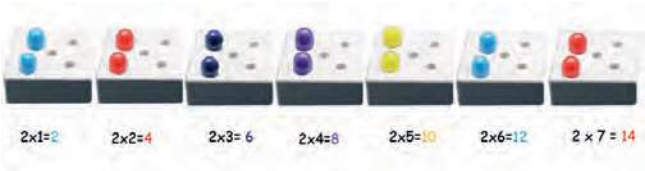




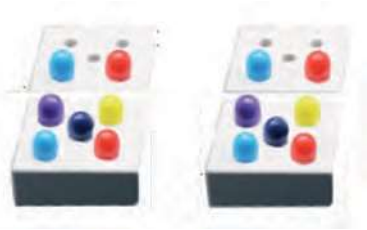
In a subsequent lesson, the student explored double 8. They partitioned to double (linked to finger patterns) then linked repeated addition to multiplication, so 8 + 8 or 8 × 2 = 16. Next, they chose to put the pegs into groups of four and noticed 4 × 4 also = 16. Finally, they chose to check their answer by grouping in 5s as they knew from previous experience that 16 was one more than three fives! Through the use of the dice dot pattern structure the maths was easily exposed, not only enabling this child to develop and connect ideas independently but also to think flexibly.

Carol

The pegs and boards can be used in the same way to explore multiplicative reasoning and connections and the boards can also be moved to demonstrate division. The use of five colours below highlights the pattern in the ones digit:



In this example, one peg from each group of two above is placed onto two sets of boards to develop an understanding of commutativity. The above shows seven groups of two and below shows two groups of seven:



This representation of double 7 is easily conceptually subitised as the boards have partitioned 7 into an iconic 5 and 2. The total can also be regrouped as 10 + 4.

Seeing children confidently partition numbers to work out new multiplication facts after working with Spot On is very pleasing. I was delighted that a girl I was working with, who had exhibited weak number sense two months previously, used this decomposition of 7 to work out 6 × 7:

She said that she knew 6 × 5 = 30
and $\underline{6 \times 2 = 12}$
so 6 × 7 must be 42

6 × 7 could also be linked to 3 × 7:

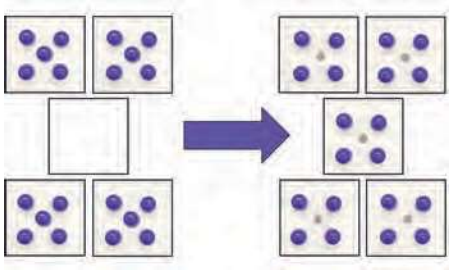
3 × 7 = 21
$\underline{3 \times 7 = 21}$
so 6 × 7 = 42

All these explorations provide strong foundations for flexible thinkers. Many children may turn to written methods for multiplication for a calculation such as 24 × 7, but if they are encouraged to see the links in their multiplication facts, they are more likely to see that they can do this calculation mentally by linking it to 12 × 7.

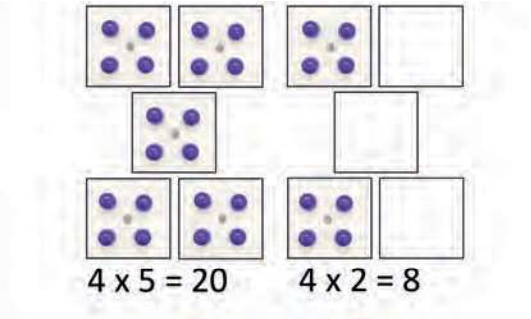
Carol

An EFF funded study found that using dice dot patterns offered a useful means of exposing the structure of multiplication. The study on 'Using patterns to model multiplication', working with lower attaining students found that dice patterns helped move students on from repeated addition and students gained a genuine insight into the laws of commutativity (Küchemann, Hodgen and Konstantine, 2016).

The study explored commutativity as four groups of five arranged in the iconic dice arrangement, which was then rearranged as five groups of four. In this picture we are demonstrating this by removing the peg at the centre of each five and creating a new group of four in the centre of the pattern:

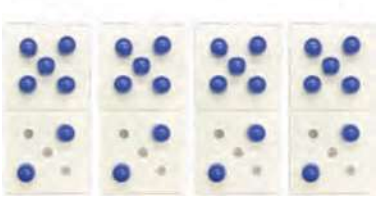


This particular multiplication mat is a very powerful representation for exposing the structure of multiplication as both the multiplicand and multiplier is easily subitised and links can be explored. For example, 4 × 7:



If children have physically explored the commutativity, the five groups of four is easily connected to a multiplication fact that the child is confident with.

Using dice patterns creates many strong visuals for multiplication where the results can be easily subitised and the links explored. Here, 7 × 4 is represented and the product can be easily subitised.



The EEF study cited previously also covers the use of arrays, which is another very powerful way of representing multiplication. It shows commutativity and can be divided to explore links. It links to the area model and can be connected to 'realistic' arrays such as a sheet of stamps. The study concluded, however, that the dice patterns were 'sufficiently different from the standard array (partitioned or not) to offer a fresh insight into multiplication' (Küchemann, Hodgen and Konstantine, p42, 2016), thereby proving to be useful variation to deepen understanding.

Louise and Carol

In our research led approach, we develop subitising and visualising skills utilising structured dot patterns, which has proved to be an effective tool to develop a deep sense of number.

We are very aware that potentially one in four children are struggling with learning maths and we know these children often lack early number sense. We have found that dot patterns offer the structural variation that children need to gain the required depth of conceptual understanding. Therefore, we would advocate integrating this effective pedagogy into mainstream classes as a foundation of good mathematics teaching, to potentially prevent some maths difficulties and ensure depth for all. Furthermore, this will not only aim to address the attainment gap but also develop all children as mathematicians with a strong sense of number, who can think flexibly, confidently make decisions and work efficiently with numbers.

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Carol Handyside is a Maths Specialist who created Spot On With Numbers out of a passion to ensure all children have firm foundations in mathematics through developing a strong sense of number: **https://www.spotonwithnumbers.co.uk**

Louise Langford is a Dyscalculia Specialist Teacher. She tutors and leads Professional Development opportunities for school improvement in Mathematics.

MATHS FUN FOR COOL KIDS

BOOK REVIEW

Maths Fun for Cool Kids

Authors: **Katie Knapman and Edward Matthews**
Publisher: **Jumping Yak, https://www.jumpingyak.com/**
ISBN 13: **978-1916101203**
Price: **£8.99**

Maths Fun for Cool Kids is a puzzle book full of curriculum-related activities. Aimed at 8 to 12 year-olds, the puzzles are presented in no particular order, indispersed with cartoons and jokes. Some pages have a 'Brain Bender!' stamp, indicating that these puzzles are a little trickier and may require some lateral thinking. Where necessary, vocabulary is briefly summarised to ensure the child can access the puzzle.

I liked the way a mix of approaches including the order of calculating, prime numbers, multiples and more was used to generate the dates of birth of famous historical figures such as Henry VIII and Shakespeare, the date of Operation Overlord and the death of Thomas Becket – and this is just one example of its quirkyness. There are some familiar problems, but they are usually phrased in a different way or the answer is given and a different question asked. These puzzles are challenging and intriguing but accessible too, giving children the opportunity to develop their fluency and reasoning as they solve non-routine problems. The problems will certainly help to keep children on their toes, but if a child gets really stuck, the answers are in the back of the book so they can work backwards to understand what the question was asking of them.

Written by Katie Knapman with one of her son's maths teachers, Edward Matthews, this book is quirky and fun. The puzzles will encourage children to be flexible in their thinking and hone their pattern spotting skills. Its original and unusual style will tempt many away from a screen and get them thinking mathematically.

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PRIMARYMATHS at the checkout on the website **www.jumpingyak.com** to get a 20% discount.

Cherri Moseley

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